# MATHMATICS SAMPLE PAPER - 2007
# CLASS XII

Time 3.15 hrs M.M.100

Additional time of 15 minutes is given for reading and understanding the question paper not writing the answers.

*General instruction:*

1. *The question paper consists of three parts. A,B and C. Section A is compulsory for all students .in addition to section A, every student has to attempt B or C part.*
2. *For section A*
   *Question 1 to 8 carry 3 marks each Question 9 to 15 are of 4 marks question 16 \ to 18 are of 6 marks each.*
3. *Section B or section C*
   *Question 19 to 22 are of 3 marks each*
   *Question 23 to 25 are of 4 marks each.*
   *Question 26 carry 6 marks.*
4. *All question are compulsory.*
5. *Internal choices have been provided in some question. You have to attempt only one of the choices in such question.*
6. *Use of calculator is not permitted. However you may ask for logarithmic & statistical tables, if required.*

## SECTION A

1. IF A = $\begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \\ 1 & 0 & 3 \end{bmatrix}$. find the inverse of A.

2. Prove that $\begin{vmatrix} 1 + a^2 - b^2 & 2ab & -2b \\ 2ab & 1 + a^2 - b^2 & 2a \\ 2b & -2a & 1 - a^2 - b^2 \end{vmatrix} = (1 + a^2 + b^2)^3$

3. A bag contains 8 marbles of which 3 are blue and 5 are red. One marble is drawn at random, its colour is noted and the marble is replaced in the bag. A marble is again drawn from the bag and its colour is noted. Find the probability that the marbles will be
   ( i ) blue followed by red. ( ii ) Blue and red in any order.
   ( iii ) of the same colour.

4. Find the probability distribution of the number of doublets in four throws of a pair of dice.

5. Find the differential equation corresponding to $y^2 = m ( a^2 – x^2 )$

OR

Solve the differential equation: $y'' – 2y + y = 0$, $y(0) = 1$, $y'(0) = 2$, $y = xe^x + ex^2$

6. Solve the differential equation: $( e^x + e^{–x} )\ y' = ( e^x – e^{–x} )$
7. Evaluate $\int 1 / \sin(x–a) \cos(x–b)\ dx$.

8. Evaluate $\int \sqrt{\tan x}\ dx$.

9. Examine the validity of the following argument:

$S_1 : [ p \wedge ( \sim q ) ] \rightarrow r$  $S_2 : p \vee q$  $S_3 : q \rightarrow p$  $S : r$

OR

Write the Boolean expression and the Boolean function given by the input / output table given below:

| **Input** | | | **Output** |
|---|---|---|---|
| $x_1$ | $x_2$ | $x_3$ | s |
| 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 0 | 1 | 1 |
| 1 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 0 |
| 0 | 0 | 0 | 1 |

10. Show that $\lim_{X \to 0} [ x ( 1 – \sqrt{( 1 – x^2 )} ) ] / [\sqrt{( 1 – x^2 )} ( \sin^{-1} x ) ] = 1 / 2$
11. Differentiate first principal $e^{\sqrt{x}}$ w.r.t x from first principal.

12. If $\sqrt{( 1 – x^2 )} + \sqrt{( 1 – y^2 )} = a ( x – y )$, then prove that $y' = \sqrt{[ ( 1 – y^2 ) / ( 1 – x^2 ) ]}$

13. Find the intervals in which the function f given by $f (x) = \sin x + \cos x$ , $0 \le x \le 2\pi$. Is increasing or decreasing.

14. Evaluate: $\int e^x [ \tan^{–1} x + 1 / ( 1 + x^2 ) ]\ dx$.

15. Evaluate: $\int_0^{\pi} dx / ( 5 + 4 \cos x )$
16. Prove that the area of a right – angled triangle is maximum when the triangle is isosceles.
b

17. Evaluate $\int_a$ cos x dx as the limit of a sum.

OR

Find the area of region included bounded by the ellipse $x^2 / 9 + y^2 / 4 = 1$.

18. Solve the following system of linear equations by matrix method.
$x - y + z = 4$; $2x + y - 3z = 0$ and $x + y + z = 2$

## SECTION – B

19. Find the scalar component of a unit vector which is perpendicular to the vectors $i + 2j - k$ and $3i - j + 2k$.

20. What is vector triple product. For any three vectors a, b and c
$\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$

21. A body is projected vertically upward with a velocity u; after time t, another body is projected vertically upward from the same point with velocity v, where $v < u$. If they meet as soon as possible, prove that
$t = [u - v + \sqrt{(u^2 - v^2)}] / g$.

22. For $[1 / p]^{th}$ of the distance between two stations, a train is uniformly accelerated and for $[1 / q]^{th}$ of the distance it is uniformly retarded. It starts at rest from one station and comes to rest at another. Prove that the ratio of the greatest velocity to its average velocity is $[1 + (1 / p) + (1 / q)] : 1$

OR

A man swims with constant velocity. He takes time $t_1$ to swim across a river in still water and time $t_2$ to cross the river when the river is flowing. If b is the width of the river, show that the velocity of the river is $[b \sqrt{(t_2^2 - t_1^2)}] / t_1 t_2$.

23. A plane meets the co-ordinate axes in A, B, C and $(\alpha, \beta, \gamma)$ is the centroid of the triangle ABC. Then show that the equation of the plane is
$x/\alpha + y/\beta + z/\gamma = 3$.

24. The angle of inclination between two forces P and Q is $\theta$. If P and Q are interchanged in position, show that the resultant will turn through an angle $\phi$, where
$\tan \phi/2 = [(P - Q) \tan \theta/2] / (P + Q)$

OR

A particle is placed at the centre of a circle and is acted upon by three forces P, Q and R tending towards the vertices A, B and C, respectively of a circumscribed triangle. Prove that when the forces are in equilibrium

$P / \cos(A/2) = Q / \cos(B/2) = R / \cos(C/2)$; where A, B, C are the angles of the triangle.

25. ABCD is a rhombus of side a and angle $\alpha$. A square iCEFD is constructed on CD, so as to lie on the side of CD away from AB. Forces P each acting along the lines AB, BC, DA, DF, FE, EC, respectively and a force 2P acts along CD. Show that the system reduces to a single couple whose moment is $2a P ( 1 - \sin \alpha )$

26. Find the equation of the plane through the line of intersection of the planes . $3x - 4y + 5z = 10$, $2x + 2y - 3z = 4$ and parallel to the line $x = 2y = 3z$.